Staff Papers Series

STAFF PAPER P84-28 September 1984

A THEORETICAL AND EMPIRICAL APPROACH
TO THE VALUE OF INFORMATION IN
RISKY MARKETS

Frances Antonovitz
Terry Roe

Department of Agricultural and Applied Economics

University of Minnesota
Institute of Agriculture, Forestry and Home Economics
St. Paul, Minnesota 55108

A THEORETICAL AND EMPIRICAL APPROACH TO THE VALUE OF INFORMATION IN RISKY MARKETS

by

Frances Antonovitz
and
Terry Roe*

* Frances Antonovitz is an assistant professor at the University of California, Davis. Terry Roe is a professor at the University of Minnesota.

Invited paper presented at the Annual Meetings of the American Agricultural Economics Association, Cornell University, August 5-8, 1984.

A THEORETICAL AND EMPIRICAL APPROACH TO THE VALUE OF INFORMATION IN RISKY MARKETS

ABSTRACT

The theory of the competitive firm under price uncertainty is used to develop a money metric of a producer's willingness to pay for additional information. This concept is extended to the market by formulating ex-ante and ex-post measures of the value of a rational expectations forecast. The empirical feasibility of these measures are demonstrated by application to a simple two equation econometric model of an agricultural market.

I. INTRODUCTION

The central focus of this paper is to develop an easily computable money metric of an agent's willingness to pay for information under risk, to extend this concept to the market, and to demonstrate its application in a simple two equation econometric model of the U.S. fed cattle market. The paper draws on previous contributions to the theory of competitive firm under price uncertainty, namely Rothschild and Stiglitz, Sandmo, and more recently Pope (1978, 1980) and Pope, Chavas and Just. The latter contributions provide insight into the econometric application of the theory and into the validity of producer surplus measures of firm welfare under risk. In these models, the production decision is made given the producer's subjective distribution of output price. In this context, the value of information to an individual agent can be formulated using a Bayesian approach which amounts to a comparison of expected utility levels from choices based on prior information with choices based on additional information. Contributions in this area are numerous and include those of Lindley, Winkler, and more recently Gould and Hess who focused on the effects of risk preferences and the nature of the distribution of random events on the value of information. The approach developed in this paper departs from the Bayesian method by omitting the step where agents update their prior distribution However, the assumption that agents form a subjective distribution and that the value of information can be based on a comparison of utility levels is maintained.

The conceptual approach presented in this paper facilitates empirical application. For a restricted class of utility functions, it's shown that the money metric of an agent's willingness to pay for additional information can be computed from the firm's risk averse supply or factor demand function. While other studies (e.g., Hayami and Peterson, Freebairn, DeCanio) have derived welfare estimates of the value of a forecast, the approach here is the first, to our knowledge, to incorporate agents' risk preferences in a market level econometric model and to estimate the value of information as a function of the mean and variance of a rational expectations forecast.

The problem is specified in section II followed by the conceptual framework for measuring an individual's willingness to pay for additional information and the value of information to the market in section III. To illustrate the approach, an econometric model is specified in section IV; and the results from fitting it to time series data from the U.S. fed cattle market are reported in section V. The empirical results suggest (a) that producers are risk averse, (b) that the bimonthly mean value of information to a typical producer varies from a deflated 12 cents per cwt to 41 cents per cwt over the 1970-80 period depending on the amount of additional information, and (c) that the mean bimonthly expected (ex-ante) value of a rational expectations forecast to the market is about 21 cents per cwt with periodic gains and losses to both producers and consumers.

II. THE PROBLEM

The competitive firm under price uncertainty is described in a Bernoullian framework where the agent's expected utility function is a strictly concave, continuous and differentiable function of profits. In this case the primal-dual function can be expressed as

$$L^* = EU[Pq^* - C(q^*)] - EU[Pq - C(q)] \tag{1}$$

where the first and second bracketed terms are the indirect and direct expected utility function respectively, P is stochastic output price, C(q) is the cost function and E is the expectation operator. The first order equation for a minimum is the familiar condition

$$\frac{\partial L^*}{\partial q} = -E[\left(P-c'(q)\right) U'(\pi)] = 0 \qquad (2)$$

where $U'(\pi) = dU/d\pi$ and $C'(q)$ is positive and continuous.

To describe the different output choices that occur when the agent's distribution of output price is based on different sets of information and to facilitate the derivation of various measures of the value of information, two states of information are defined: the subjective and the more informed state.

Let $f^o(p)$ denote the agent's distribution of output price in the subjective state based on the information utilized by the agent at the time the output decision is made. The optimal quantity of output in the subjective state can be determined by solving equation (2) for q where the expectation, denoted E^o, is taken with respect to $f^o(p)$. The agent's optimal output choice will be represented by q^o. However, prior to the realization of output price, profit is a stochastic variable and can be expressed as

$$\pi^o = Pq^o - C(q^o).$$

The utility in the subjective state that the agent expects to obtain from producing q^o is $E^oU(\pi^o)$.

In the more informed state the agent's beliefs are based on more information than is embodied in $f^o(p)$. Let $f^m(p)$ denote this more informed distribution of output price which, while not fundamental to our approach, can be viewed as having the properties of a Bayesian posterior distribution obtained from updating $f^o(p)$ with additional data such as an independently supplied price forecast. The optimal output choice in the more informed state, denoted by q^m, can again be determined by solving equation (2) for q with expectations, E^m, taken with respect to the more informed distribution. Prior to realization of the output price, profit is a stochastic variable represented by

$$\pi^m = Pq^m - C(q^m).$$

The utility in the more informed state that the agent expects to obtain from producing q^m is $E^m U(\pi^m)$.

The first problem is to derive an easily computable money metric of an agent's willingness to pay for the additional information embodied in the more informed distribution $f^m(p)$. When the output response resulting from the adoption of additional information by a group of agents alters market price, the more informed distribution must embody the price effects of this response. These issues are dealt with in the next section.

III. THE VALUE OF INFORMATION

The Value of Information to an Individual Producer

Two different measures of the value of information to an individual agent are presented. The first is an ex-ante measure. In this case, decisions made based on information embodied in the prior $f^o(p)$ are compared with those made in the more informed state with information embodied in $f^m(p)$. The second measure is a special case of the first; it is a measure of the value of perfect information, determined by comparing realized profits from the choice q^o with profits obtained when price is known with certainty.

First, consider the difference in expected utility in the more informed state between production choices q^o and q^m. The maximization of $E^o U(\pi)$ yields the optimal quantity q^o with corresponding expected utility $E^oU(\pi^o)$ in the subjective state. However, the expected utility of the choice q^o in the more informed state is $E^mU(\pi^o) = E^m[U(Pq^o - C(q^o))]$. Hence, the value of information, in utility units, can be defined to be the difference in the more informed state between the expected utility of producing q^m and the expected utility of producing q^o:

$$VI_1 = E^mU(\pi^m) - E^mU(\pi^o). \tag{3}$$

It can be shown that VI_1 will always be non-negative. Consider the primal-dual problem stated in equation (1). Since $EU(\pi^*)$ is the maximum value of expected utility that can be attained over all possible values of profit,

$$L^* = EU(\pi^*) - EU(\pi) \geq 0.$$

By derivation of quantity q^m, it is clear that $q^m = q^*$ in equation (1) when expectations are taken with respect to $f^m(p)$. Hence,

$$E^mU(\pi^m) - E^m U(\pi) \geq 0$$

for all values of π. Thus, VI_1 is non-negative.

This measure of the value of information is not very useful because utility has only ordinal properties. To avoid this problem, a money metric similar to equivalent variation in the certainty case can be derived.

Using equation (3), define a nonstochastic variable VI_2 such that

$$E^mU(\pi^m) = E^mU(\pi^o + VI_2).\underline{1/} \tag{4}$$

To show that VI_2 is non-negative, recall that $U'(\pi) > 0$ implies $U(\pi_1) > U(\pi_2)$, if $\pi_1 > \pi_2$. Since it has already been shown from the primal-dual problem that $E^mU(\pi^m) \geq E^mU(\pi^o)$, then by equation (4), $E^mU(\pi^o + VI_2) > E^mU(\pi^o)$. By definition of expectations,

$$\int U(\pi^o + VI_2)f^m(p)dp > \int U(\pi^o)f^m(p)dp. \tag{5}$$

But by the properties of integrals, expression (5) implies $U(\pi^o + VI_2) \geq U(\pi^o)$ for all p. Since $U'(\pi) > 0$, $\pi^o + VI_2 \geq \pi^o$. And hence, VI_2 is non-negative. The value of VI_2 is the amount of money that must be given to the agent when he produces q^o so that his expected utility is the same as if he had produced q^m.

The empirical advantage of this approach lies in the ease of obtaining a money metric of the value of having the additional information embodied in $f^m(p)$. In general, knowledge of the agent's utility function and $f^m(p)$ are required to compute the value of information. However, knowlege of the initial beliefs $f^o(p)$ are not required. Estimates of $f^m(p)$ may come about through public or private price forecasts or research that yields insights into factors determining the distribution of P.

The usefulness of this approach is enhanced if the expected utility function is restricted to a member of the following class:2/

$$EU = E\pi + g(q,\vec{\sigma}); \quad \vec{\sigma} = (\sigma_2, \sigma_3, \ldots, \sigma_k) \tag{6}$$

where σ_k represents the kth central moment of price. It has been shown by Pope and others that the indirect expected utility function corresponding to (6) is related to the risk averse supply function as follows

$$\frac{\partial EU(\pi^*)}{\partial EP} = q^*. \tag{7}$$

Pope, Chavas, and Just show that if equation (7) holds, producer surplus, given by the area behind the risk averse supply curve, is a money metric of utility.

To derive an explicit expression for VI_2, the supply function in the more informed state can be stated as $q^m = q(p\ ,\ \vec{\sigma}^m)$. Then, VI_2 is given by

$$VI_2 = \int_{p_{q^o}}^{\bar{p}^m} q(p\ ,\ \vec{\sigma}^m)dp - q^o \cdot (\bar{p}^m - p_{q^o}) \tag{8}$$

where the lower limit of integration is the value of p_{q^o} satisfying the expression $q^o = q(p_{q^o}, \vec{\sigma}^m)$. To show that this condition is the money metric VI_2, it follows from (6) that expanding (8) yields

$$\begin{aligned} VI_2 = {} & \bar{p}^m q^m - C(q^m) - g(q^m, \vec{\sigma}^m) \\ & - \bar{p}^m q^o + C(q^o) + g(q^o, \vec{\sigma}^m) \end{aligned} \tag{9}$$

which is precisely condition (4) when expected utility is of the form (6).[3/]

This result is depicted in Figure 1. If the agent's subjective distribution of output price results in a production level of $q_1^{\,o}$, the value of information is given by the triangular area a. Area b depicts the value of information when the optimal output choice in the subjective state is $q_2^{\,o}$. Empirical estimates of these values for the fed cattle industry appear in a later section of this paper.

A measure of the value of perfect information is a special case of equation (8). With perfect information, the agent's subjective distribution degenerates to the nonstochastic realized price p^r. If the price p^r had been known before the production decision was made, the utility maximizing choice of output is that amount which maximizes profit. Let q^* denote this optimal level of production and $\pi^* = p^r q^* - C(q^*)$ be the corresponding maximum profit. However, output choice q^o which maximizes expected utility based on $f^o(p)$ yields realized profit of $\pi^r = p^r q^o - c(q^o)$. The value of perfect information can be determined from equation (9).

$$VI_2 = p^r q^* - C(q^*) - p^r q^o - C(q^o) = \pi^* - \pi^r.$$

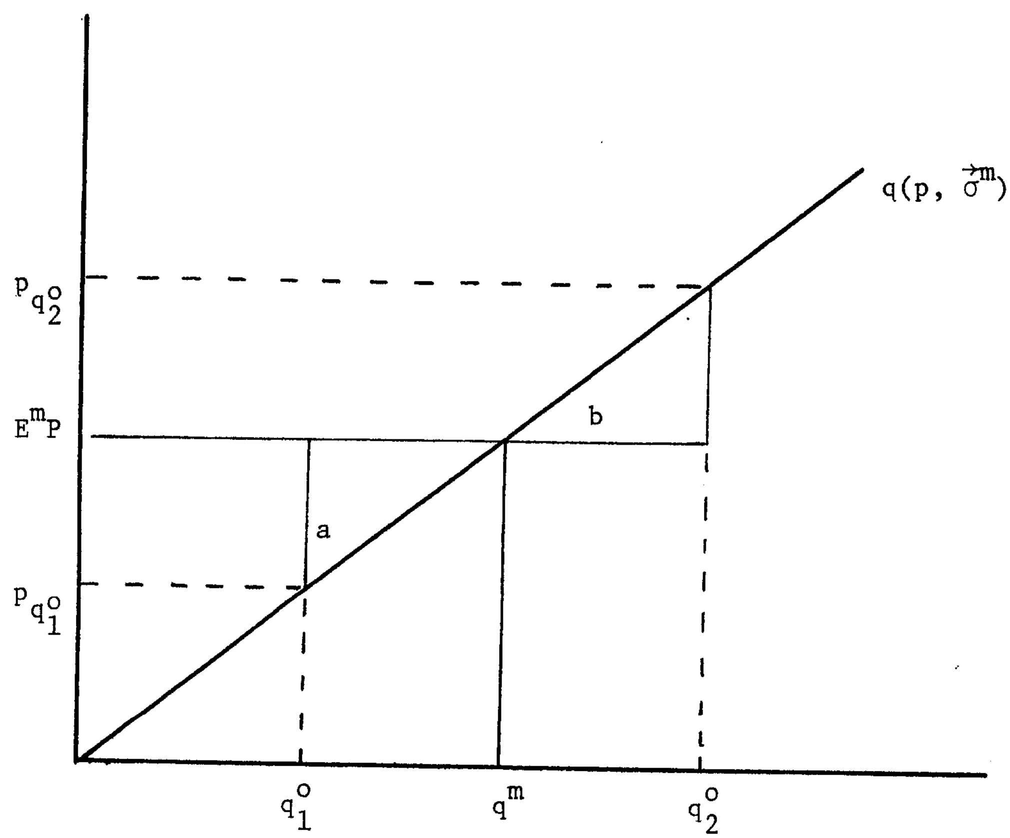

Figure 1. Value of Information to an Individual Agent

The graphical analysis of the value of perfect information is similar to Figure 1 except that the risk averse supply function is replaced by the traditional supply function with no variance term.

The Value of Information to the Market

A money metric of the value of information to an individual producer whose production decisions have no influence on market price has now been presented. To determine the value of information to both producers and consumers in a market, it is necessary to consider market supply and demand functions. Also, additional information may influence the market supply curve because the adoption of additional information by the producers in forming a more informed distribution of output price may result in a shift in the market supply curve. Certainly, a more informed forecast of market price and the variance of this price must incorporate these shifts in supply. In this paper, the value of a rational expectations forecast of the mean and variance of fed cattle price which are provided by perhaps a public agency are considered. The key assumptions employed to obtain measures of the value of information based on the rational expectations forecasts developed in this section are: (i) all producers are identical so there is no aggregation problem in deriving market supply, (ii) the rational expectations forecast is given as a distribution of output price, (iii) all producers in the market adopt the forecast as their more informed distribution, (iv) all exogenous variables whose values are unknown at the time the forecast is formulated are assumed to follow stable stochastic processes, and (v) the agency providing the forecast is assumed to know the parameters of the model.[4/]

For ease of exposition, it will be assumed that the distribution of output price can be expressed in terms of its first two moments. From assumption (i), the farm level market supply in time period t can be states as

$$Q_t^s = S(E^m(P_t), \sigma_t^m) \tag{9}$$

where $\sigma_t^m = E^m(P_t - E^m(P_t))^2$, $Q_t^s = Nq_t$, and N denotes the number of producers in the industry. The expressions $E^m(P_t)$ and σ_t^m are the first two moments of the distribution of market price. Expectations are taken with respect to the more informed distribution $f^m(p)$ formulated in a time period previous to t which, by assumption (ii), is based on the rational expectations forecast, defined below. The exogeneous variables normally appearing in equation (9) are omitted for convenience since their values are assumed to be known when producers make production commitments.

Let

$$P_t = D(Q_t^d, Z_t) \tag{10}$$

denote the inverse farm level demand function where Q_t^d denotes the quantity demanded and Z_t denotes a vector of random exogeneous variables.

The model is closed by assuming that in each period the price equilibrates quantity demanded and quantity supplied. Thus, market price can be determined by using equations (9) and (10).

$$P_t = D[S(E^m(P_t), \sigma_t^m), Z_t]. \tag{11}$$

It has been assumed that the public forecast is a rational expectations forecast; hence, expected price is determined by taking the conditional expectation of equation (11). Depending on the form of the supply and demand functions, the rational expectations forecast can be stated as

$$E^m(P_t) = F(\sigma_t^m, E(\vec{Z}_t)) \ . \tag{12}$$

where σ_t^m is defined by equation (13) and $E(\vec{Z}_t)$ is the expected value in a period previous to t of the vector of exogeneous variables which, by assumption (iv), follow stable stochastic processes.

The rational expectations variance of market price is

$$\sigma_t^m = E^m(P_t - E^m(P_t))^2. \tag{13}$$

The quantity supplied can be determined by substituting equations (12) and (13) into (9) which gives

$$Q_t^s = S(F(\sigma_t^m, E(\vec{Z}_t)), \sigma_t^m) = S(G(\sigma_t^m, E(\vec{Z}_t)). \tag{14}$$

Next, ex-ante and ex-post measures of the value of a rational expectations forecast will be developed. Rational expectations market equilibrium is depicted in Figure 2. Expected demand, or the expectation of equation (10), is represented by ED; and expected supply, or equation (14), is given by QS. If producers had not been supplied with a rational expectations forecast, let Q_t^o be the quantity they would have produced and $E(P_t^o)$ be the expected price associated with Q_t^o based on the expected demand curve. The quantity Q_t^o is obtained from the supply curve (9) where the moments of the output price are based on the producers' subjective distribution $f^o(p)$ rather than the more informed rational expectations distribution.

Marshallian consumer surplus is used as an approximate money metric for changes in consumers' utility although it's well known shortcomings are recognized. The value of a rational expectations forecast is estimated by measuring the changes in consumers' and producers' surplus arising from the output choices based on the two alternative distributions $f^o(p)$ and $f^m(p)$. Similar to measuring the value of information to an individual agent, producers' surplus is determined from the risk averse supply curve (14).

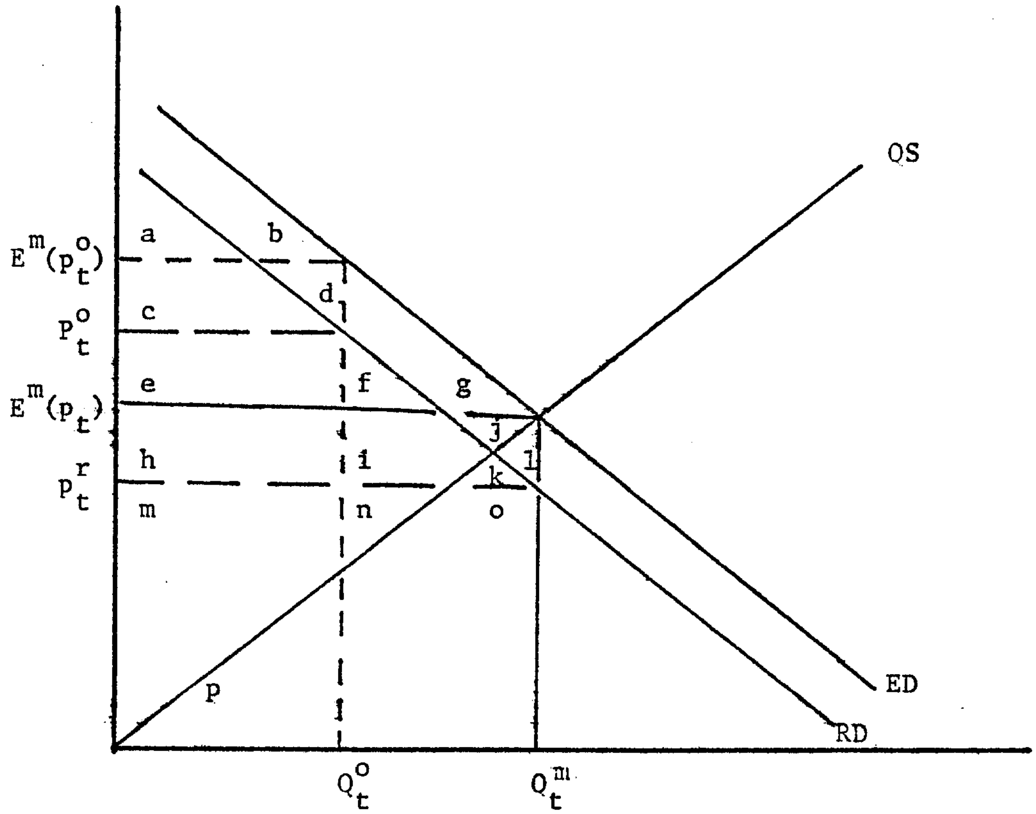

Figure 2. Value of Information Under Rational Expectations Forecasts

The expected value in the more informed state of the change in consumers' surplus, denoted by $E^m \Delta CS_t$ when producers base production decisions on $f^m(p)$ instead of $f^o(p)$ is given in Figure 2 by the area

$$E^m \Delta CS_t = c+d+e+f+g. \tag{15}$$

For the individual producer, area a in Figure 1 measured the producer's willingness to pay for the information embodied in $f^m(p)$. However, when considering the value of a rational expectations forecast to an entire market, the adoption of $f^m(p)$ by all producers induces a decline in expected price from $E(P_t^o)$ to $E^m(P_t)$. This corresponds to a decline in producers' surplus, denoted $E^m \Delta PS_t$, equivalent to the area

$$E^m \Delta PS_t = i+j+n - (c+d+e). \tag{16}$$

The expected dead weight loss (the ex-ante market value of information), denoted $E^m VI^{re}$, is the triangle

$$E^m VI_t^{re} = f+g+i+j+n \tag{17}$$

Note that unlike the situation for the individual producer depicted in Figure 1, it is possible for either consumers or producers, but not both, to suffer a welfare loss from a more "accurate" distribution of market price.

Next, an ex-post measure of the value of information will be considered which is useful in empirical application because it provides insights into the accuracy of the ex-ante measure just presented. The ex-post measure of the value of information is the realized value of information to the market which can be determined after the exogeneous random variables in the demand equation are observed.

Denote the vector of observed values of the demand function by $\vec{Z}_t^r$. In Figure 2, RD represents the demand curve with $\vec{Z}_t^r$. The realized prices from production choices Q_t^o and Q_t^m are depicted as p_t^o and p_t^r, respectively.

The ex-post value of the change in consumers' surplus when producers based production decisions on $f^m(p)$ instead of $f^o(p)$ are given in Figure 2 by the area

$$\Delta CS_t = e+f+h+i+k. \tag{18}$$

Similarly, by using prices p_t^o and p_t^r, the ex-post change in producers' surplus is given by

$$\Delta PS_t = n - (e+h+k+l) \tag{19}$$

and the ex-post value of information to the market is

$$VI_t^{re} = f+i+n-l. \tag{20}$$

This value can be negative depending on the magnitude of the triangle l which is determined by the realizations of the random variables.

More generally, the ex-ante value of information to the market ($E^m VI_t^{re}$) can be stated as

$$E^m VI_t^{re} = \int_{Q_t^o}^{Q_t^m} (D(Q_t, E\vec{Z}_t) - S^{-1}(Q_t, \sigma_t^m))dQ_t. \tag{21}$$

The lower and upper limits of integration are the quantities supplied to the market in the absence and presence of a rational expectations forecast, respectively. The second bracketed term in the expression is the inverse of the supply function (14). Similarly, the ex-post value of information to the market can be stated as

$$VI_t^{re} = \int_{Q_t^o}^{Q_t^m} (D(Q_t, \vec{Z}_t^r) - S^{-1}(Q_t, \sigma_t^m))dQ_t. \tag{22}$$

The results presented in the remaining sections of the paper are based on equations (9) to (14), (21) and (22).

IV. EMPIRICAL FRAMEWORK

The expected utility function for an individual agent, equation (6), does not provide much insight into the functional form of the indirect utility function because π depends on, among other factors, the underlying production function. For notational convenience, let V denote the form of the indirect utility function. The procedure employed here is to approximate V by a second order Taylor series expansion.

Let the parameters of V be represented as the vector $\vec{W} = (\vec{p}_1, \bar{p}, \vec{\sigma})$ where $\vec{p}_1$ is a vector of n input prices. When all the parameters have been normalized around their mean values, expanding V around $\vec{W} = \vec{0}$ yields

$$V = V(\vec{0}) + \sum_{i=1}^{\ell} \frac{\partial V(\vec{0})}{\partial W_i} W_i + 1/2 \sum_{i=1}^{\ell} \sum_{j=1}^{\ell} \frac{\partial^2 V(\vec{0})}{\partial W_i \partial W_j} W_i W_j + \text{higher order terms.} \tag{23}$$

Truncate the expression at the second order and substitute the following terms:

$$V(\vec{0}) = \alpha_o; \ \frac{\partial V(\vec{0})}{\partial W_i} = \alpha_i; \ \frac{\partial^2 V(\vec{0})}{\partial W_i \partial W_j} = \beta_{ij}.$$

Hence,

$$V(\vec{W}) \simeq \tilde{V}(\vec{W}) = \alpha_o + \sum_{i=1}^{\ell} \alpha_i W_i + 1/2 \sum_{i=1}^{\ell} \sum_{j=1}^{\ell} \beta_{ij} W_i W_j. \tag{24}$$

By Young's theorem, there is symmetry between cross partial derivatives. Thus, $\beta_{ij} = \beta_{ji}$. Let $W_i = p_{1i}$ where i = 1, 2 ..., n; $W_{n+1} = \bar{p}$; and $W_{s+n} = \sigma_s$ where s = 2, 3, ..., k. From the partial derivatives of (24) with respect to W

$$\frac{\partial \tilde{V}}{\partial W_{n+1}} = \alpha_{n+1} + \sum_{j=1}^{\ell} \beta_{n+1,j} W_j. \tag{25}$$

By condition (7) equation (25) can be expressed as

$$q^* = \alpha_{n+1} + \sum_{j=1}^{\ell} \beta_{n+1,j} W_j + R \tag{26}$$

where R is a residual due to the truncation of the Taylor series at the second order.

By assumption (i), equation (26) can be multiplied by a factor of N to obtain the market supply curve

$$Nq^* = N\alpha_{n+1} + \sum_{j=1}^{\ell} N\beta_{n+1,j} W_j + NR$$

or

$$Q^s = a + \sum_{j=1}^{\ell} b_j W_j + \xi \tag{27}$$

Recall that it has been assumed that the distribution of output price can be expressed in terms of its first two moments. Hence, for the example of the fed cattle industry presented here, $\vec{w}$ is a vector of input prices and the mean and variance of fed cattle price. A more explicit form of the supply function can be written

$$Q_t^s = a + \sum_{i=1}^{n} b_i P_{1i(t-1)} + c\overline{P}_t + d\sigma_t + \varepsilon_t \tag{28}$$

where ε_t includes error due to the truncation of the Taylor series. For our purposes here, the farm level demand for fed beef was specified in a linear price dependent form as

$$P_t = e + fQ_t^d + \sum_{j=1}^{z} g_j Z_{jt} + v_t \tag{29}$$

where v_t is the error term. Expressions for the mean and variance of a rational expectations forecast can now be determined. Equilibrium in the market, equation (11), requires that supply equals demand or

$$P_t = e+f(a + \sum_{i=1}^{n} b_i \overline{P}_{1i(t-1)} + cP_t + d\sigma_t + \varepsilon_t) + \sum_{j=1}^{z} g_j Z_{jt} + v_t \tag{30}$$

Taking conditional expectations of both sides and rearranging yields the following expression, which is equivalent to equation (12), for the expected price of a rational expecations forecast

$$E^m(P_t) = \frac{e+f(a+\sum_{i=1}^{n} b_i P_{1i(t-1)}+d\sigma_t^m) + \sum_{j=1}^{z} g_j E(Z_{jt})}{1-c} \tag{31}$$

From equation (29), a rational expectations estimate of the variance can be determined.

$$E^m\sigma_t = \sum_{j=1}^{z} g_j^2 \, E\big(Z_{jt} - E(Z_{jt})\big)^2 + f^2\sigma_\varepsilon^2 + \sigma_v^2 \quad \underline{5/} \tag{32}$$

V. EMPIRICAL RESULTS

Estimates of the Supply and Demand Functions

Aggregate bimonthly data on cattle slaughter for the period from the second bimonth of 1970 to the fifth bimonth of 1980 were used to estimate supply and demand equations (28) and (29). The input prices included in the supply equation were feeder cattle, corn, and soybean meal. However, the subjective variables $\bar{p}$ and σ are not observable. Hence, before equation (28) can be fit to data, an auxiliary model must be formulated as an analogue of producers' forecasts to obtain instsruments for $\bar{p}$ and σ. It is not assumed that producers were using rational expectations forecasts of the mean and variance of market price. An ARIMA (2, 1, 0) model was used to estimate, three to four bimonths in advance, the mean and variance of the aggregate subjective distribution of fed cattle price, $f^o(p)$. The ARIMA model was used, in part, because Bessler found that the ARIMA model gave the best estimates of the moments of aggregate subjective distributions on yield. However, it is recognized that conditional forecasts obtained by using other models may have provided a better fit of the total supply equation to the data. When the supply function was fit to data, first order autocorrelation in the disturbance terms was observed. Hence, a modified Cochrane-Orcutt procedure was used to obtain a maximum likelihood estimate of the autocorrelation coefficient and the data was transformed accordingly.

Empirical estimates of the parameters of equation (28) appear in Table 1. Overall, the supply function fits the data remarkable well. Coefficient estimates on the price of corn and feeder cattle are significant and of the expected sign. The soybean meal price coefficient is not significantly different from zero indicating perhaps that soybean meal is not an extensively used input for cattle feeding in the United States. Important for our purposes here is the significance and expected signs of the coefficients on the ARIMA forecast of mean and variance of cattle price. These results suggest that the supply function is upward sloping and that fed cattle producers are risk averse.

The exogeneous variables used to estimate the demand equation included per capita disposable income and a farm level index of other meats. Parameter estimates of the bimonthly farm level demand for fed cattle appear in Table 2. Due to the evidence of first order autocorrelation, the procedure followed to transform the data and to obtain parameter estimates was the same as the procedure used to fit the supply function to data. Overall, the linear price dependent demand function appears to fit the data reasonably well although the coefficient associated with per capita disposable income is not significantly different from zero at the .05 level. Important for our purposes here is the significance and expected sign of the coefficient associated with the quantity of fed cattle demanded.

Table 1. Parameter Estimates of the Market Risk Averse Supply Function of Fed Cattle Production, Bimonthly from 1970 to 1980.

Independent Variables	Coefficient Estimates
Constant	92,235,000.**
Corn Price	-7,522,300.*
Soybean Meal Price	16,428.
Feeder Cattle Price	-1,501,900.**
Mean Fed Cattle Price	1,041,700.**
Variance of Fed Cattle Price	-481,220.*

R^2 is .81

First order autocorrelation coefficient is .45606

Variance of the estimate corrected for first order autocorrelation is 5.0921 X 10^{12}

* Indicates significance of a two-tailed t-test at the .05 percent level.

** Indicates significance of a two-tailed t-test at the .01 percent level.

The corn price was the average price received by farmers in Iowa. Soybean cake and meal price, 44 percent protein, bulk in Decatur was used. Feeder cattle price was determined by averaging 400-500 pound and 600-700 pound choice feeder steers in Kansas City. All input prices were divided by the USDA's index of prices paid by farmers. The ARIMA forecasts of the mean and variance are of the deflated average fed cattle price received by farmers in the U.S. Estimates of fed cattle production were obtained from the USDA's bimonthly commercial cattle slaughter. All prices were in 1972 dollars.

Table 2. Parameter Estimates of the Inverse Market Farm Level Demand Function of Fed Cattle, Bimonthly from 1970 to 1980.

Independent Variables	Coefficient Estimates
Constant	2.973×10
Quantity of Fed Cattle	3.0895×10^{-7}**
Per Capita Disposable Income	6.2023×10^{-3}
Farm-Level Index of Other Meats	2.798×10*

R^2 is .86

First order autocorrelation coefficient is .84449

Variance of the estimate corrected for first order autocorrelation is 2.6776

* Indicates significance of a two-tailed t-test at .05 percent level.

** Indicates significance of a two-tailed t-test at the .01 percent level.

Estimates of fed cattle production were obtained from the U.S.D.A.'s bimonthly commercial cattle slaughter. Per capita disposable income and average fed cattle price received by farmers in the U.S. were deflated to 1972 dollars. The farm level index of other meats was determined as follows

$$I_t = \frac{\sum P_{it}Q_{it}}{\sum P_{it}Q_{it} + P_{Bt}Q_{Bt}}$$

where the P_i and Q_i are the farm level prices and quantities of chicken and pork, and P_B and Q_B are for beef.

<u>The Value of Information to an Individual Producer</u>

Using equation (9) along wtih the parameter estimates reported in Table 1, estimates of the value of informaton are obtained from simulations based on two more informed distributions of fed cattle prices. These distributions are hypothetical because they are not based on additional analysis or composite forecasts of the fed cattle price series. They are a more accurate description of fed cattle prices in the sense that for each bimonth the mean price and variance values selected are closer to the realized price than is the ARIMA forecast.[6/]

Bimonthly estimates of the value of information for the two hypothetical distributions mentioned above are reported in Table 3 for the years 1978 to 1980. Descriptive statistics of the value of information estimates for the entire period 1970-1980 are reported at the bottom of the table. The fed cattle price and the corresponding ARIMA forecast and variance are also reported. For 1978 through the fourth bimonth of 1979, the ARIMA model generally underestimated price and for the remainder of the period, fed cattle price was overestimated. The variance of the forecast increased over the period.

The results indicate that for a single producer or a group of producers whose output levels have no noticeable effect on market price, the value of information embodied in distribution (D-I) (with the more informed mean 50 percent closer to the realized price than the ARIMA mean and with more informed and ARIMA variances equal) averages about 12 cents per cwt over the entire period and ranges from a low of nearly zero to a high of 97 cents per cwt. The value of information embodied in an even more accurate forecast (D-II) (with more informed mean equal to the realized price and more informed variance only one half of the ARIMA variance) averages about 20 cents per cwt for the entire

Table 3. Bimonthly Estimates of the Value of Information of Two More Accurate, Though Hypothetical, Distributions of Fed Cattle Price and the Value of Perfect Information in Deflated Dollars Per Cwt Live Weight of Fed Cattle Produced. 1978-1980

	Fed Cattle Price Realized	ARIMA Forecast		Value of Information		Value of Perfect Information
		Mean Price	Variance	D-I	D-II	
78-1	20.818	18.905	11.649	.0255	.2261	.5018
78-2	23.915	18.800	11.614	.1618	.7528	1.2141
78-3	26.145	18.725	11.587	.1517	.8938	1.3886
78-4	25.204	19.600	11.724	.2339	.9391	1.4585
78-5	26.336	22.100	12.334	.0200	.3275	.6716
78-6	26.301	24.805	13.396	.0010	.0943	.3360
79-1	30.325	25.535	13.923	.0383	.4930	.9846
79-2	34.234	25.895	13.757	.9665	2.8427	3.9603
79-3	32.211	26.575	13.846	.4366	1.5116	2.3465
79-4	29.027	28.620	14.341	.4666	.2954	.7260
79-5	29.281	32.700	15.579	.0011	.0207	.2312
79-6	28.050	33.300	16.064	.0318	.0058	.0749
80-1	27.850	30.910	15.856	.0969	.0129	.0509
80-2	25.923	29.970	15.825	.0051	.0071	.1879
80-3	24.731	29.260	15.793	.0499	.0087	.0650
80-4	25.653	28.225	16.080	.0006	.0407	.3000
80-5	24.554	27.085	16.241	.0799	.0036	.0757
Mean (1970-1980)				.1146	.2002	.4135
Std. Deviation				.1735	.4840	.6652
Minimum				.0006	.0002	.0002
Maximum				.9665	2.8427	3.9603

a/ D-I denotes a hypothetical distribution of fed cattle price where the bimonthly mean is 50 percent closer to the bimonthly realized price than is the (2, 1, 0) ARIMA forecast; but identical variance. D-II denotes a hypothetical distribution of fed cattle price where the bimonthly mean is equal to the realized bimonthly price and the variance is 50 percent of the (2, 1, 0) ARIMA forecast variance.

period, ranging from approximately zero to a high of \$2.84 cents. The high occurred in the second bimonth of 1979 which serves to point out that the value of information is larger the greater the difference between q^o and q^m. q^o will tend to be smaller than q^m when $\bar{p}^o$ is smaller than $\bar{p}^m$ and when σ^o is larger than σ^m. q^o will tend to be larger than q^m when the opposite relationships occur between the parameters of the subjective and more informed distributions.

The value of perfect information appears in last colume of Table 3. The estimated mean value of perfect informaton is about 41 cents per cwt although the range in value is from approximately zero per cwt to 3.96 per cwt. Again, the largest value of information occurred in the same year as the previous case, a year when forecast price was low and the variance of forecast price was relatively high.

These results are of limited usefulness in addressing the welfare implications of a forecast when product demand is downward sloping. The results for this case are presented next.

The Value of Information to the Market

The estimates of the value of information for an individual producer presented in the previous section are based on a hypothetical more informed distribution. The moments of a more informed distribution for an entire market are theoretically based on the rational expectations conditions given by equations (12) and (13) and are empirically based on equations (31) and (32).[7/] The estimates of the ex-ante and ex-post value of information to the market are based on equations (21) and (22), respectively. The results of this analysis appear in Table 4.

The mean and variance of the rational expectations forecast appear in columns four and five of Table 4. Contrasting these variance estimates with

Table 4. Bimonthly Estimates of the Value of a Rational Expectations Forecast in Deflated Dollars Per CWT Live Weight of Fed Cattle Produced, 1978-1980.

	Fed Cattle Price Realized	Expected Realized Price	Realized Rational Expectations Price	Rational Expectations Fo-ecast Mean	Rational Expectations Fo-ecast Variance	Ex-Ante Value Of Information	Ex-Post Value Of Information
	(P_t^o)	$(E^m(P_t^o))$	(P_t^r)	$(E^m(P_t))$	(σ_t^m)	$(E^m VI_t^{re})$	(VI_t^{re})
78-1	20.818	23.581	19.409	22.173	11.179	.1896	-.0348
78-2	23.915	24.074	22.063	22.222	11.189	.3338	.2442
78-3	26.145	24.154	24.593	22.602	11 198	.2328	.2866
78-4	25.204	24.903	23.201	22.901	11.225	.3969	.2994
78-5	26.336	24.673	25 705	24.041	11.230	.0383	.0543
78-6	26.301	25.762	26.095	25.556	11.248	.0042	.0011
79-1	30.325	27.301	29.650	26.626	11.257	.0472	.1221
79-2	34.234	29.557	31.573	26.895	11.244	.8271	1.4472
79-3	32.211	29.397	30.216	27.401	11.145	.4540	.7121
79-4	29.027	29.174	28.025	28.171	11.212	.1111	.0761
79-5	29.281	29.547	29.226	29.492	11.210	.0003	-.0015
79-6	28.050	29.605	28.233	29.788	11.179	.0038	.0226
80-1	27.850	29.450	28.133	29.733	11.255	.0088	.0330
80-2	25.923	30.649	24.930	29.656	11.237	.1146	-.1222
80-3	25.731	29.893	24.090	29.252	11.230	.0464	-.1236
80-4	25.653	29.510	24.543	28.400	11.213	.1363	-.0322
80-5	24.554	27.856	24.259	27.561	11.189	.0089	-.0190
Mean (1970-1980)						.2113	.1470
Std. Deviation						.2994	.3111
Minimum						.0003	-.3654
Maximum						1.4568	1.4472

the variance of the ARIMA forecast in column three of Table 3 shows that the rational expectations forecasts exhibit smaller variances. A comparison of columns three and four of Table 4 shows the relationship between the rational expectations forecast $E^m(P_t)$ and realized price when all producers are assumed to adopt the forecast p_t^r; this is depicted in Figure 2. The difference in these prices are attributable to the values obtained by the random variables in the model.

The expected realized price $E^m(P_t^o)$ which appears in column 2 of Table 4 is the price that is expected to prevail if producers do not adopt the forecast. (See Figure 2). The closer the expected realized price is to the rational expectations forecast of price, the closer Q_t^m is to Q_t^o; and hence, the smaller the value of information. Additional results not reported in the table suggest that the adoption of a rational expectations forecast will have only a small effect on fed cattle production. For the period from 1970 to 1980, the mean of the expected bimonthly production when all producers are assumed to adopt the rational expectations forecasts is 98.32 percent of the mean bimonthly production levels actually produced.

The expected value of information to the market given by equation (21) appears in the sixth column of Table 4. The mean expected bimonthly value of information for the 1970-1980 period was $.21 per cwt of production or, in total value terms, a mean of approximately $13.3 million per bimonth. The expected value of information ranges in value between virtually zero to a maximum of $1.47 per cwt; the minimum value occurred in the fifth bimonth of 1979 while the maximum value occurred in the second bimonth of 1974. The expected value of information increases when the ARIMA forecasts diverge from the rational expectations forecast depending on the relative variances of the forecasts.

This divergence tends to occur at turning points in the ARIMA price series where prices differ from those of the rational expectations price series.

Not appearing in the table are the ex-ante expected producer and consumer gains and losses from adopting the rational expectations forecasts which are given by equations (18) and (19). The bimonthly mean value of information to producers of fed cattle during 1970 to 1980 was estimated to be \$.49 per cwt, ranging from a minimum of \$-2.04 to a maximum of \$4.76. On average, consumers lose \$.28 per cwt from the adoption of the rational expectations forecast by producers. The range in consumer gains and losses is from a high of \$2.86 to a low of \$-3.35. Expected gains to producers are positive when the expected quantities produced under the rational expectations forecast are less than the quantities actually produced. The converse relationship holds for expected gains to consumers.

As indicated by equation (22), the ex-post value of information in the market is obtained after the random variables in the supply and demand curves have been observed. These ex-post values, appearing in the last column of Table 4, provide insights into the validity of the ex-ante estimates of the value of information. The mean bimonthly value of this ex-post measure is approximately \$.15 per cwt, ranging from \$-.37 to \$1.45. Of the 64 bimonthly estimates obtained, negative values, although small, were reported 36 percent of the time. Hence, the adoption of the forecast by producers would have resulted in a "realized" welfare loss to the market 36 percent of the time. Nevertheless, the gains still outweighed these losses.

The mean of the bimonthly estimates of the ex-post value of information to producers average \$.43 per cwt., a small decline from the above reported figure for the ex-ante value of information. The range varies from a high of

\$4.15 per cwt. to a low of \$-1.68 per cwt. The loss to consumers was virtually unchanged at \$0.29 per cwt., with a range of \$2.77 per cwt. to \$-3.28 per cwt.

VI. SUMMARY AND CONCLUSIONS

An easily computable money metric of a risk averse agent's willingness to pay for additional information was developed and extended to the market in this paper. The procedure was empirically demonstrated for a restricted class of utility functions by fitting a risk averse supply function and a farm level demand function to time series data from the U.S. fed cattle industry. While, in our view, this paper makes a contribution to methods for estimating the value of information, numerous restrictive assumptions were employed; and numerous hurdles remain before reliable empirical estimates can be obtained on the informational efficiency of markets. For instance, our approach does not take account of an agent's updating of information along Bayesian lines, nor is the cost of information acquisition and processing included in the conceptual framework. While the empirical framework was only developed to illustrate the application of the conceptual framework, it nevertheless serves to illustrate both the strengths and weakness of this approach. A significant weakness is the restrictions that must be placed on the class of utility functions for empirical purposes. Consideration of both price and production risk can also further complicate the empirical model, and the specification of an empirical framework as an analogue of agents' expectation formation process is particularly troublesome. Nevertheless, the empirical analysis gives plausible results. The empirical results suggest that agents are risk averse, that the expected value of an improved forecast does increase producer utility, and that the expected market value of information is of empirical magnitudes that are plausible. These results lend to the feasibility and credibility of further experimentation with this approach.

FOOTNOTES

1/ Lindley (1971) describes a similar measure for the value of information, Z, given by $E^{\beta}[U(\pi^{\beta} - Z)] = E^{\beta}[U(\pi^{o})]$, where expectations are taken with respect to the Bayesian posterior distribution $f^{\beta}(p)$. Although both Lindley's Z and our VI_2 are measures of the amount the agent is willing to pay to obtain more information, in general they may not be equal. There is also a subtle difference in interpretation. In the Bayesian approach Z is the amount of money which must be given up by the agent when he produces q^{β} so that he has the same amount of utility in the more informed state as producing q^{o}. In our case, VI_2 is the amount of money that must be given to the agent when he produces q^{o} so that his expected utility in the more informed state is the same as if he had produced q^{m}. See Roe and Antonovitz for a graphical analysis of this and other measures of a money metric when $f^{m}(p)$ has only two parameters.

2/ Antonovitz estimated a supply function for the fed beef industry employing the assumption that agents' utility functions were members of the class given by equation (6). The data failed to reject this hypothesis.

3/ The value VI_2 can also be obtained from the risk averse factor demand function, $-\partial EU(\pi^*)/\partial C = X^*$, where C is the price of input X, in a manner analogous to (9).

4/ Frydman suggests that a rational expectations equilibrium is possible only if agents know the true parameters of the model or that prices converge to a rational expectations equilibrium if agents have consistent estimates of the parameters. Hence, assumption (v) guarantees that a rational expecations equilibrium exists.

5/ It is assumed that the endogeneous variables are independent and that the true values of the parameters are known.

6/ This is similar to Freebairn's choice of a more accurate forecast of the mean.

7/ The mean and variance of the stochastic processes of the exogeneous variables of the demand function were estimated using a moving ARIMA model. For the income variable, an ARIMA (1, 1, 0) was used. For the index variable, an ARIMA (2, 1, 2) was used.

REFERENCES

Antonovitz, Frances. "The Value of Information and Allocative Error Under Risk in Fed Cattle Production: The Role of Cash and Futures Markets." Ph.D. dissertation, University of Minnesota, 1982.

Bessler, David A. "Aggregated Personalistic Beliefs on Yields of Selected Crops Using ARIMA Processes." American Journal of Agricultural Economics 62(November 1980): 666-674.

DeCanio, Stephen J. "Economic Losses from Forecasting Error in Agriculture." Journal of Political Economy 88(April 1980): 234-258.

Freebairn, J.W. "The Value and Distribution of the Benefits of Commodity Price Outlook Information." Economic Record 52(June 1976): 199-212.

Frydman, Roman. "Towards an Understanding of Market Processes: Individual Expectations, Learning, and Convergence to Rational Expecations Equilibrium." American Economic Review 72(September 1981): 652-668.

Gould, John P. "Risk, Stochastic Preference, and the Value of Information." Journal of Economic Theory 8(1974): 64-84.

Hayami, Yujiro and Willis Peterson. "Social Returns to Public Information Services: Statistical Reporting of U.S. Farm Commodities." American Economic Review 62(March 1972): 119-130.

Hess, James. "Risk and the Gain from Information." Journal of Economic Theory 27(1982): 231-238.

Lindley, Dennis. Making Decisions. London: Wiley-Interscience, 1971.

Pope, Rulon D. "The Expected Utility Hypothesis and Demand-Supply Restrictions." American Journal of Agricultural Economics 62(November 1978): 619-627.

_______. "The Generalized Envelope Theorem and Price Uncertainty." International Economic Review 21 (February 1980): 75-86.

Pope, Rulon D., Jean-Paul Chavas, and Richard Just. "Economic Welfare Evaluations for Producers Under Uncertainty." American Journal of Agricultural Economics 65(February 1983): 98-107.

Roe, Terry and Frances Antonovitz. "Willingness to Pay for Information Under Price Uncertainty: Theory and Application." Department of Agricultural and Applied Economics, University of Minnesota, Working Paper P84-16, 1984.

Rothschild, M. and J. Stiglitz. "Increasing Risk: II. Its Economic Consequences." Journal of Economic Theory 3(1971): 66-84.

Sandmo, Agnar. "On the Theory of the Competitive Firm Under Price Uncertainty." The American Economic Review 61 (March 1971): 65-73.

Winkler, Robert L. Introduction to Bayesian Inference and Decision. New York: Holt, Rinehart and Winston, Inc. 1972.